P. MATHS.
w. MECH. II

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UNIVERSITIES OF MANCHESTER LIVERPOOL
LEEDS SHEFFIELD AND BIRMINGHAM

JOINT MATRICULATION BOARD

GENERAL CERTIFICATE OF EDUCATION

PURE MATHEMATICS WITH MECHANICS (O 26) PAPER II

ORDINARY

Friday 28 June 1963 2—4

Negligently presented or slovenly work will be penaIized.
Mathematical tables (green covers) and two sheets of graph paper will be provided

Answer **six** *questions.*

[Take *g* as 32 ft. per sec.^{2} or as 980 cm. per sec.^{2}]

[Turn over

1. (*a*) The sides *AB, BC* of the rectangle *ABCD* are of lengths 3*a* ft., 4*a* ft. respectively. Forces of magnitudes *P* lb. wt., 2P lb. wt., *P* lb. wt. act along the lines *AB, A C, A D* respectively. Find the magnitude of the resultant of these forces. This resultant meets *BC* at X. Find *BX.*

When an additional force *Q* acts along *AD,* the resultant now acts along *AC.* Calculate the ratio *P:* Q.

(*b*) A uniform rod *HK* of weight *W* lb. wt. is smoothly hinged to a fixed pivot at *H* and is maintained in equilibrium in a horizontal position by a light string attached to *K.* This string makes an angle of 30^0 with *HK* produced. Find the tension in the string. Find also the magnitude and direction of the resultant force exerted by the pivot on the rod.

2. (a) The sides of a uniform square lamina *ABCD* are each of length 12 in. and *X*, *Y* are the mid-points of the sides *AB, BC* respectively. The triangular portion *XBY* is removed. Calculate the distances of the centre of gravity of the remaining pentagon *AXYCD* from *AX* and *AD.* This pentagon is suspended from a smooth fixed pivot at *A* and hangs freely under gravity. Calculate, correct to the nearest degree, the inclination of *AD* to the downward vertical.

(*b*) A car of mass 27 cwt. stands on a horizontal road. The distance between the points of contact of the front and rear wheels with the ground is 9 ft. The centre of gravity of the car is 4 ft. measured horizontally from the front axle. Calculate the magnitude of the resultant force exerted on the road by the two rear wheels.

3. A non-uniform ladder *AB,* of weight *5 W* lb. wt. and length *5a* ft., rests in limiting equilibrium with *A* on rough horizontal ground and *B* against a rough vertical wall. The centre of gravity of the ladder is at G, where *AG* = 2a ft. The coefficient of friction at each end of the ladder is $\frac{1}{2}$. Calculate, correct to the nearest minute, the angle made by the ladder with the vertical. Show that the total reaction at the wall is perpendicular to the total reaction at the ground and calculate, in terms of *W,* the magnitudes of these total reactions.

4. (*a*) Two ships *A* and *B* each leave the same port at noon. Ship *A* steams South at 10 m.p.h. and ship *B* steams West at 24 m.p.h. Find the velocity of *B* relative to *A*.

The greatest distance at which the ships can communicate by short-wave radio is 100 miles. Find, correct to the nearest minute, the latest time at which this means of communication can be used.

(*b*) A ball is projected horizontally with speed *u* ft. per sec. and *t* sec. later the line joining the ball to the point of projection makes an angle θ with the horizontal. Prove that $u = \frac{1}{2} gt \cot\theta$

5. (*a*) A rough horizontal turntable rotates about a vertical axis and describes 40 complete revolutions per minute. A particle of mass 2 kilogram is placed on the turntable at a distance of 30 cm. from the axis and it does not slip on the turntable. Calculate, in kilogram wt., the force of friction between the particle and the table. Hence find, correct to two significant figures, the least coefficient of friction possible between the particle and the table.

(*b*) A railway truck of mass 10 tons, moving along a straight level track with a speed of 6 m.p.h., collides with and is immediately coupled to a stationary truck of mass 12 tons. At the same instant the brakes of both trucks are applied, locking the wheels so that the trucks slide. The coefficient of friction between a wheel and a rail is $\frac{1}{4}$. Calculate, in ft., the distance moved by the trucks before they come to rest.

6. To the ends *A, B* of a light string are attached particles of masses 3 lb., *5* lb. respectively. The string passes over a small smooth pulley fixed at the highest point of a fixed wedge which has two smooth faces each inclined at 30° to the horizontal and one particle lies on each of these two faces. The system is released from rest with the string taut and in a vertical plane containing lines of greatest slope of the sloping faces. Calculate, in ft. per sec.^{2}, the acceleration of each particle. Calculate also, in lb. wt., the tension in the string and the resultant force exerted by the string on the pulley.

Two seconds after motion commences the string is cut. Calculate the total distance moved by the lighter particle before it first comes to instantaneous rest.

[Turn over

7. A cable-car of mass $2\frac{1}{4}$ tons is pulled up a slope of inclination , θ where $\sin\theta = \frac{1}{14}$ against a constant frictional force of 195 lb. wt. Calculate, in lb. wt., the tension in the towing cable when the acceleration of the car is $\frac{2}{3}$ ft. per sec.^{2} up the slope.

The car starts from rest and moves with this constant acceleration. Calculate the speed of the car after 40 sec. and show that the engine winding the cable is then working at the rate of 32 horse-power.

(1 horse-power = 550 ft. lb. wt. per sec.)

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