# University of Nottingham

FACULTY OF PURE SCIENCE

SCHOOL OF PHYSICS

PART II EXAMINATION, 1968

PHYSICS (vi)

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SATURDAY *June* 8*th* 9.45 - 12.45

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*Answer* THREE *questions from section* A, ONE *question from section* B *and* ONE *question from section* C

*Candidates should note that equal marks are assigned to the three sections, and that it is intended that approximately one hour should be spent on each section*

*Use a separate answer book for each section and label each book clearly* A, B *or* C

## SECTION A

1. Two aircraft, *A* and ***B***, approach one another along a straight line. Their engines have the same frequency *f*, but owing to the Doppler effect the pilot of *A* hears a frequency 2*f*, anti the pilot of *B* a frequency of *4f.* Calculate the airspeed of each aircraft. (Velocity of sound = 700 m.p.h.)

2. Atoms *A* and *B* have fractional co-ordinates $(\frac{1}{6},\frac{1}{3},\frac{1}{2})$ and $(\frac{1}{6},\frac{1}{3},\frac{1}{2})$ in an orthorhombic unit cell whose dimensions are $a$ = 10.8 Å, $b$ = 8.4 Å, $c$ = 7.2 Å. Find the distance *AB* and the angles hetween *AB* and the cell axes.

3. In times of crisis Martians communicate by transmitting bursts of an electric field whose magnitude increases from zero as $t^2$, where $t$ is the time from the beginning of the pulse. The receiving svstem of these creatures contains an inductance of 2 units, a capacitor of ½ unit and a resistor of 4 units, in series with their aerial. If their brain monitors the charge on the condenser, evaluate without using a transform method the time dependence of the signal reaching the brain during a pulse. [Assume all charges are neutralised between pulses.]

4. The equation $\tanh x = \lambda x$ has a solution $x = 0$ for all λ. Investigate its other solutions as λ is varied. What are the approximate solutions when λ = 0.5 ?

5. A spiral spring is used as a solenoid. Calculate the change in its length when a current is passed through it, in terms of the geometry of the solenoid and the spring constant $k$ per unit length. Neglect the change in radius.

6. Find the radius of the largest spherical atom which can fit into an interstice in α-iron: α-iron can be regarded as a body-centred cubic structure with cube side $a$ consisting of spherical iron atoms in contact.

7. A multicore cable consists of eight identical wires. Four two-terminal components have to be connected at each end of the cable to randomly selected pairs of wires so as to form a continuous circuit. What is the probability that such a circuit will be obtained?

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SECTION B

8. (*a*) Fourier analyse the function *f*(*x*) defined by:

$$f(x)=\begin{cases} x & (0\le x\le \frac{1}{2}a) \\ a-x & (\frac{1}{2}a\le x\le a.\end{cases}$$

(*b*) Using the method of separation of variables, obtain a solution to the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$$

for $0\le x\le a$ and which satisfies the following boundary conditions:

(i) $V\to 0$ as $y\to\infty$
(ii) $V = 0$ when $x = 0$and $x = a$ for all $y\ge 0$,
(iii) $V = f(x)$ as obtained in (*a*) when $y = 0$.

9. (*a*) Give a *brief* summary of the use in physics of Laplace's equation and indicate qualitatively how physically valid solutions may be obtained in both two and three dimensions.

(*b*) Show that, if $W=U+iV$ is an analytic function of the complex variable $z=x+iy$ where $U,V,x$ and $y$ are real and $i^2=-1$, then $U$ and $V$ both satisfy Laplace's equation in two dimensions.
In one particular example,

$$U=\frac{x}{x^2+y^2}$$

Find *W* in terms of *z* and sketch the contours in the *z*-plane which correspond to $U$ = constant and $V$ = constant. Describe a physical problem to which your solution could be applied.

(*c*) How do Legendre polynomials, $P_n(\cos\theta)$, arise in physical problems? Use Rodrigues' formula,

$$P_n(\mu)=\frac{1}{2^n n!}\frac{d^n}{d\mu^n}(\mu^2-1)^n,$$

to find the first four ($n$ = 0, 1, 2, 3) Legendre polynomials.

(*d)* The function *f(x)* is defined by

$$f(x)=\begin{cases} x & (0\le x\le \frac{1}{2}a) \\ a-x & (\frac{1}{2}a\le x\le a.\end{cases}$$

By using a procedure similar to that used in Fourier analysis, determine the value of the coefficients $A_n$ for $n$ = 0, 1, 2, 3 if *f*(*x*) is written in the form

$$f(x)=\sum_{n=0}^{\infty}A_nP_n(x)$$

[You may assume that

$$\int_{-1}^{1}P_m(x)P_n(x)dx=\frac{2}{2n+1}\delta_{mn}.$$

10. (*a*) Both analytic and non-analytic functions occur in mathematical physics. Explain what is meant by these functions in the above statement and give two examples of each, taken from physics. Name two techniques which may be used to simulate non-analytic functions by analytic functions.

(*b*) Given that the Laplace transform of $f_1(t)$ is

$$F(p)=\int_0^{\infty}\exp(-pt)f_1(t)dt,$$

show that the Laplace transform of the function $\phi(t)$, where

Question continued at top of next page

$$\phi(t) = \begin{cases} 0 & (0 \le t \le a) \\ f_1(t-a) & (t > a) \end{cases}$$

is equal to $\exp(-ap)f(p)$ for $a > 0$.

(*c*) If $f_2(t)$ is a periodic function, of period $T$, show that

$$\int_0^\infty \exp(-pt) f_2(t)dt = \frac{1}{1-\exp(-pT)} \int_0^T f_2(t)\exp(-pt)dt .$$

(*d*) Find the Laplace transform of a 'saw-tooth' function of period $T$ which has a maximum amplitude of unity and a minimum amplitude of zero.

## SECTION C

11. Describe how you would

(a) measure the velocity of a bullet moving at approximately 500 m sec$^{-1}$ ,

(*b*) measure the thickness of a metal film known to be about 2 x $10^{-4}$ cm thick which is deposited on a flat glass plate,

(*c*) demonstrate that **B** = 0 inside a superconductor.

Estimate the accuracy of measurement in each case.

12. A directly heated thermionic diode consists of a cathode made from a single filament of metal wire with a cylindrical anode. The diode characteristics ( $I_a$ in mA and $V_a$ in V) together with supplementary information about the filament current $I_f$, its resistance $R_f$ and its temperature $T_f$ are given in the following table:

| | | | | | | |
|---|---|---|---|---|---|---|
| $I_f$ (A) | 0.84 | 0.88 | 0.92 | 0.95 | 0.97 | 0.99 |
| $R_f$ (Ω) | 4.75 | 4.96 | 5.13 | 5.25 | 5.35 | 5.43 |
| $T_f$ (°K) | 1680 | 1730 | 1770 | 1800 | 1830 | 1844 |

| $I_a$ | $V_a$ | $I_a$ | $V_a$ | $I_a$ | $V_a$ | $I_a$ | $V_a$ | $I_a$ | $V_a$ | $I_a$ | $V_a$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.28 | 4.0 | 0.28 | 4.0 | 0.28 | 4.0 | 0.28 | 4.0 | 0.28 | 4.0 | 0.28 | 4.0 |
| 0.91 | 25 | 0.80 | 8.0 | 0.80 | 8.0 | 0.80 | 8.0 | 0.80 | 8.0 | 0.80 | 8.0 |
| 1.12 | 80 | 1.70 | 10 | 2.00 | 15 | 2.00 | 15 | 2.00 | 15 | 2.00 | 15 |
| 1.40 | 150 | 2.00 | 80 | 3.02 | 35 | 4.40 | 25 | 4.40 | 25 | 4.40 | 25 |
| | | 2.30 | 150 | 3.20 | 80 | 4.95 | 50 | 6.80 | 57 | 7.20 | 35 |
| | | | | 3.48 | 150 | 5.13 | 100 | 6.98 | 100 | 8.72 | 50 |
| | | | | | | 5.35 | 150 | 7.18 | 150 | 9.25 | 87 |
| | | | | | | | | | | 9.50 | 150 |

Account for the form of the characteristics and describe briefly how the measurement of the parameters given in the above table would be made.

Test the following:

(*a*) the Child-Langmuir relation

$$I_a \ \alpha \ V_a^{3/2},$$

(*b*) the Richardson-Dushman relation for the saturation emission current $I_s$

$$I_s \ \alpha \ T_f^2 \exp\left(-\frac{\phi}{kT_f}\right).$$

What other relations could be tested with the data given?

13. The wind speed at a site is recorded at intervals of one minute. The daily, weekly and monthly maximum, minimum and mean pressures are required, and for more detailed examination the data should be in the form of histograms for each day, week and month. Draw the flow chart and explain the logic of a computer program suitable for analysing the data and extracting all the required information.

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